Simple Harmonic Motion – Chapter 10

Recall, Hooke's Law for an ideal spring:

$$F = kx$$

- fig. 10.1 and 10.2 on p. 276

  sign convention: stretch + F and + x, compression – F and – x

The shorter the spring, the larger its spring constant.

$$k \propto \frac{1}{\#\text{ coils}}$$

- conceptual example 2 on p. 277

By Newton's 3rd law, if a force is exerted on a spring, the spring must exert a force back on the object pushing or pulling the spring. This reaction force is called the <u>restoring force</u>.

Hooke's Law Restoring Force on an Ideal Spring:

$$F = -kx$$

<u>Simple Harmonic Motion</u> (Harmonic means repeating)

Simple Harmonic Motion (SHM) is oscillatory motion that occurs when a restoring force acts on an object (its position vs. time graph should be sinusoidal).

- fig. 10.4, 10.5, 10.6, and 10.7 on p. 278

Uniform circular motion is a good model for SHM, but it is not SHM. UCM is periodic motion (repeats itself at regular intervals).

- conceptual question #4 on p. 298
- p. 280 – 283

SHM is described by a cosine graph (in general form):

T = period = time for one cycle (in this case 2π)
f = 1/T (Hz)
A = amplitude (max. displacement from rest position)

SHM can also be described by displacement, velocity, and acceleration.

In the previous graph:

ex. An object that follows the SHM of a cosine curve has a period of 2 seconds and an amplitude of 50 cm. What is its position at (a.) 0.75 s and (b.) 1 s?

Velocity in SHM, which is always changing, can be found by:

The velocity varies between max. (at x = 0) and min. (where v = 0).

In SHM, v is not constant, therefore there must be acceleration:

Like velocity, acceleration varies.

Recall, U_s is the elastic potential energy stored in a spring:

$$U_s = \frac{1}{2}kx^2$$

Also, recall energy is a conserved quantity.

SHM of a Spring

An essential criterion for SHM is that a restoring force acts to restore an object to its original position. This force increases as the displacement increases.

$$F \alpha x$$

The total energy of the system depends on the amplitude.

$$E_{Total} \alpha A$$

Period of Oscillation of a Mass on a Spring:

Note: Amplitude (A) does not affect the period.

ex. A 1 kg mass is hung from a spring (k = 100 N/m). What is the period of oscillation when it vibrates?

The Pendulum

A pendulum is not true SHM, but very close if the angle the string makes is small.

- fig. 10.20 on p. 288

Period of Oscillation of a Pendulum:

Note: Amplitude or mass of bob do not affect the period.

ex. What is the period of a 40 cm pendulum?

Damped Harmonic Motion – decrease in amplitude due to dissipation of energy

Driven Harmonic Motion – increase in amplitude due to addition of energy by a “driving force” (in same direction as v)

Resonance:
Resonance is when the frequency of the driving force matches the natural frequency of vibration of an object. The amplitude will increase without limit if there is no damping.

ex. tall buildings, move back and forth in tub in synchronism to waves, Tacoma bridge

SHM Overview

- oscillatory motion which has a restoring force acting on object
- graph is sinusoidal in shape (general form is cosine function)
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If represented by a cos (general form) graph:

If represented by a sin graph: