Unit 5 – Circular Motion

Uniform circular motion, UCM, refers to an object moving in a circle (or part of a circle) at a constant (or uniform) speed.

If the object is moving at constant speed, its speed can be calculated from:

$$s = \frac{d}{\Delta t}$$

If we use once around the circle, the distance will be the circumference of the circle. ($d = 2\pi r$)

The time to go once around the circle is called the period in circular motion. ($\Delta t = T$)

$$s = \frac{d}{\Delta t} = \frac{2\pi r}{T}$$

$$v = \frac{2\pi r}{T}$$

where: v = speed or tangential velocity (m/s)
r = radius (m)
T = period (s)

In uniform circular motion, the magnitude of the velocity stays constant, but the direction is constantly changing. The direction of the velocity at any given point is found using a tangent line to the circle at that point, thus tangential velocity.

Inertia states that an object wants to stay at a constant velocity. Constant velocity means at a constant speed in a straight line. An object can change velocity, and thus accelerate, by changing speed or by changing its direction.

An object moving in uniform circular motion is constantly changing direction which means that it is constantly changing velocity.

Centripetal Acceleration

Centripetal acceleration, a_c, is the acceleration of an object in circular motion.

Centripetal means center seeking. Centripetal acceleration is always towards the center of the circle.

Centripetal acceleration, ac, can be found by:

$$a_c = \frac{v_t^2}{r}$$

where: a_c = centripetal acceleration (m/s^2)
v_t = tangential velocity or speed (m/s)
r = radius (m)

ex.
A car turns a curve at a speed of 20 m/s. If the radius of curvature of the turn is 100 m, what is the centripetal acceleration of the car?

ex.
A ball on a string swings in a horizontal circle of radius 2.0 m. If the centripetal acceleration is 15 m/s^2, what is the speed of the ball?

Centripetal acceleration can also be found using period:

ex.
Mercury moves in an approximately circular path around the sun at an average distance of 5.8×10^{10} m with a period of 8×10^{6} seconds. What is the centripetal acceleration of Mercury as it orbits the sun?

Period is the time for once around the circle in circular motion. Frequency, f, is the number of times around the circle in one second. The two are reciprocals of each other.

$$f = \frac{1}{T}$$

The two can be distinguished by their units, period is in seconds and frequency is the number of revolutions per second, or Hertz (Hz).

This will give another method of finding centripetal acceleration, using frequency:

ex.
What is the centripetal acceleration of a stone being whirled in a circle at the end of a 1.5 m string if it makes 5 revolutions every 4 seconds?

Circular motion can be described with the following formula:

$$v = \frac{2\pi r}{T}$$

$$f = \frac{1}{T}$$

$$a_c = \frac{v_t^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$